

QCD anomalies in hadronic weak decays

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Abstract

We consider the flavour-changing operators associated with the strong axial and trace anomalies. Their short-distance generation through penguin-like diagrams is obtained within the QCD external field formalism. Standard-model operator evolution exhibits a suppression of anomalous effects in K and B hadronic weak decays. A genuine set of dimension-eight $\Delta S = 1$ operators is also displayed.

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1 Introduction

Broken symmetries have often proved as useful as exact symmetries to describe the physical world. In this respect, particular attention has been paid to the anomalies of Quantum Field Theory [1]. Their presence in global axial [2] and scale [3] symmetries of massless QED and QCD is notably required by the physics of light (pseudo-)scalar particle decays into two photons [4] and hadronic transitions between quarkonium levels [5].

In hadronic weak decays, the QCD axial and trace anomalies have already been advocated to understand the unexpectedly large branching fractions for $B \rightarrow \eta' X_s$ [6] and the $\Delta I = 1/2$ selection rule in $K \rightarrow \pi\pi$ [7][8], respectively. However, to our knowledge, no real attempt has been made so far to generate *both* pseudoscalar and scalar anomalous gluonic configurations from short-distance QCD corrections to a four-quark flavour-changing effective interaction.

At lowest order in the momentum expansion, the local anomalous operators in request consist of a flavour-changing quark bilinear coupled to the colour-singlet gluon density $G^{\mu\nu}\tilde{G}_{\mu\nu}$ in the axial anomaly case and $G^{\mu\nu}G_{\mu\nu}$ in the trace anomaly one. At the one-loop level, they are thus generated by gluonic penguin-like diagrams with a heavy virtual flavour. In this paper, we infer the corresponding QCD evolution of an arbitrary four-quark operator from the propagator of the heavy intermediate flavour plunged in external gluon fields and expanded in inverse powers of the mass. The contributions of the leading anomalous operators are then displayed.

Due to our assumptions of a four-quark initial operator and a heavy intermediate flavour, the method turns out to be particularly well-suited to the estimation of short-distance charm effects in Kaon decays within the Standard Model. Yet, the insight gained allows us to clarify the role of heavy quark generated anomalies in B decays too. Besides, the formalism developed here is applicable to any four-quark effective interaction induced by physics beyond the Standard Model.

A by-product of our method is the consistent derivation of a complete set of dimension-eight operators in standard-model hadronic K decays. Contrary to ref.[9], the central role granted here to the heavy flavour propagator immersed in QCD external fields allows us indeed to avoid a misuse of the quark classical equations of motion.

2 Heavy quark induced penguin-like operators

Let us start with the computation of the one-loop penguin-like short-distance QCD corrections to the generic four-quark operator

$$Q = (\bar{q}_1 \Gamma^A q_h)(\bar{q}_h \Gamma^B q_2), \quad (1)$$

under the assumption of a heavy intermediate flavour q_h . The symbols $\Gamma^{A,B}$ stand for products of Dirac matrices. The quark bilinears are colour-singlets.

In the path integral formalism, these corrections may be obtained by functional integration of the heavy mode q_h in the presence of a classical gluonic background [10]:

$$\exp i \int d^4x \mathcal{L}_{eff} = \int \mathcal{D}q_h \mathcal{D}\bar{q}_h \exp i \int d^4x (\bar{q}_h \not{P} q_h - M \bar{q}_h q_h + g_w Q) \quad (2)$$

with g_w , the effective coupling associated with the weak operator Q , and M , the heavy flavour mass. The covariant derivative acting on spinor fields is defined by

$$P_\mu = i\partial_\mu + g_s A_\mu \quad (3)$$

with $A_\mu = A_\mu^a \lambda^a / 2$, the external gluon field and g_s , the QCD coupling constant. The Gell-Mann matrices λ^a are normalized such that $\text{tr} \lambda^a \lambda^b = 2\delta^{ab}$ ($a, b = 1, \dots, 8$).

The Gaussian integral (2) is readily performed. It is given by the functional determinant of the matrix

$$\mathcal{A}_{xy}^{ij} = \left(\mathcal{P}_x^{ij} - M \delta^{ij} + g_w \Gamma^B q_{2,x}^i \bar{q}_{1,x}^j \Gamma^A \right) \delta^{(4)}(x - y). \quad (4)$$

Colour indices are denoted by i, j ($i, j = 1, 2, 3$) and spinor indices are understood. Using the identity $\det \mathcal{A} = \exp \text{tr} \ln \mathcal{A}$ and keeping only the $\mathcal{O}(g_w)$ term, we obtain the (non-local) effective Lagrangian $\mathcal{L}_{eff}^{g_w}$:

$$\int d^4x \mathcal{L}_{eff}^{g_w} = -ig_w \int d^4x \bar{q}_{1,x}^j \Gamma^A \left(\frac{1}{\mathcal{P} - M} \right)_{xx}^{ji} \Gamma^B q_{2,x}^i. \quad (5)$$

Consequently, the central object needed to compute the local anomalous operators in request is the propagator in external field, $(\mathcal{P} - M)_{xy}^{-1} \equiv \langle x | (\mathcal{P} - M)^{-1} | y \rangle$, at $x = y$.

We will need the following formal identity:

$$\left(\frac{1}{\mathcal{P} - M} \right)_{xx} = \int \frac{d^4p}{(2\pi)^4} \frac{1}{\not{p} + \not{\mathcal{P}}_x - M}. \quad (6)$$

Intuitively, it results from the minimal substitution $p_\mu \rightarrow p_\mu + P_\mu$ in the momentum-space free-quark propagator. The above expression is in fact properly defined by the infinite series:

$$\left(\frac{1}{\mathcal{P} - M} \right)_{xx} = \sum_{n=0}^{\infty} S_n \quad (7)$$

with

$$S_n = \int \frac{d^4p}{(2\pi)^4} \frac{1}{\not{p} - M} \left((-\not{\mathcal{P}}) \frac{1}{\not{p} - M} \right)^n. \quad (8)$$

Let us emphasize that the derivatives in S_n will only affect the involved gluon fields after momentum integration, not the external states $q_{1,2}$. Indeed, the ‘plunged’ propagator is a function, not an operator. Eq.(7) will thus eventually turn into a well defined expansion in $1/M$ provided the gluonic background fluctuations are small.

We will now compute the first few S_n ’s in the gauge-invariant dimensional regularization scheme. The above integrals can be cast into the form

$$S_n = \int \frac{d^d p}{(2\pi)^d} \frac{\mu^\varepsilon N_n}{(p^2 - M^2)^{n+1}} \quad (9)$$

with $\varepsilon = 4 - d$ and μ , the regularization scale. The numerators N_n are written in a form suitable for integration using exclusively the Dirac matrices anticommutation relations

$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$. In the cases $n = 5$ and $n = 4$, we also make use of the following decompositions of γ -strings on the Clifford basis [11], after momentum integration:

$$\begin{aligned}\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} &= g_{\mu_1\mu_2}\gamma_{\mu_3} - g_{\mu_1\mu_3}\gamma_{\mu_2} + g_{\mu_2\mu_3}\gamma_{\mu_1} + i\varepsilon_{\mu_1\mu_2\mu_3}\gamma^\beta\gamma_5 \\ \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4} &= g_{\mu_1\mu_2}g_{\mu_3\mu_4} - g_{\mu_1\mu_3}g_{\mu_2\mu_4} + g_{\mu_1\mu_4}g_{\mu_2\mu_3} - i\varepsilon_{\mu_1\mu_2\mu_3\mu_4}\gamma_5 \\ &\quad + i(g_{\mu_1\mu_2}\sigma_{\mu_4\mu_3} + g_{\mu_1\mu_3}\sigma_{\mu_2\mu_4} + g_{\mu_1\mu_4}\sigma_{\mu_3\mu_2} \\ &\quad + g_{\mu_2\mu_3}\sigma_{\mu_4\mu_1} + g_{\mu_2\mu_4}\sigma_{\mu_1\mu_3} + g_{\mu_3\mu_4}\sigma_{\mu_2\mu_1})\end{aligned}\quad (10)$$

where $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ and $\varepsilon_{\mu\nu\rho\sigma}$ is the Levi-Civita tensor with $\varepsilon^{0123} = +1$. These conventions agree with ref.[12].

Gauge invariance requires S_1 to vanish. For the next few terms, we easily obtain:

$$\begin{aligned}S_2 &= \frac{-1}{(4\pi)^2}M\left(N_\varepsilon - \ln\frac{M^2}{\mu^2}\right)P_\mu P_\nu\sigma^{\mu\nu} \\ S_3 &= \frac{i}{3(4\pi)^2}\left(N_\varepsilon - \ln\frac{M^2}{\mu^2}\right)[P^\nu, [P_\mu, P_\nu]]\gamma^\mu \\ S_4 &= \frac{-i}{2(4\pi)^2}\frac{1}{M}P_{\mu_1}P_{\mu_2}P_{\mu_3}P_{\mu_4}\Gamma_4^{\mu_1\mu_2\mu_3\mu_4} \\ S_5 &= \frac{i}{2(4\pi)^2}\frac{1}{M^2}P_{\mu_1}P_{\mu_2}P_{\mu_3}P_{\mu_4}P_{\mu_5}(\Gamma_5^{\mu_1\mu_2\mu_3\mu_4\mu_5} + \Gamma_{5'}^{\mu_1\mu_2\mu_3\mu_4\mu_5})\end{aligned}\quad (11)$$

with $N_\varepsilon = \frac{2}{\varepsilon} + \ln 4\pi - \gamma$, and

$$\begin{aligned}\Gamma_4^{\mu_1\mu_2\mu_3\mu_4} &= -\frac{2}{3}g^{\mu_1\mu_3}g^{\mu_2\mu_4} + \frac{2}{3}g^{\mu_1\mu_4}g^{\mu_2\mu_3} - i\varepsilon^{\mu_1\mu_2\mu_3\mu_4}\gamma_5 \\ &\quad + ig^{\mu_1\mu_3}\sigma^{\mu_2\mu_4} - ig^{\mu_2\mu_3}\sigma^{\mu_1\mu_4} + ig^{\mu_2\mu_4}\sigma^{\mu_1\mu_3} \\ &\quad - \frac{i}{3}g^{\mu_1\mu_2}\sigma^{\mu_3\mu_4} - \frac{i}{3}g^{\mu_1\mu_4}\sigma^{\mu_2\mu_3} - \frac{i}{3}g^{\mu_3\mu_4}\sigma^{\mu_1\mu_2}\end{aligned}$$

$$\begin{aligned}\Gamma_5^{\mu_1\mu_2\mu_3\mu_4\mu_5} &= -\frac{13}{30}g^{\mu_2\mu_3}g^{\mu_4\mu_5}\gamma^{\mu_1} + \frac{17}{30}g^{\mu_2\mu_4}g^{\mu_3\mu_5}\gamma^{\mu_1} - \frac{8}{30}g^{\mu_2\mu_5}g^{\mu_3\mu_4}\gamma^{\mu_1} \\ &\quad + \frac{17}{30}g^{\mu_1\mu_3}g^{\mu_4\mu_5}\gamma^{\mu_2} - \frac{18}{30}g^{\mu_1\mu_4}g^{\mu_3\mu_5}\gamma^{\mu_2} + \frac{17}{30}g^{\mu_1\mu_5}g^{\mu_3\mu_4}\gamma^{\mu_2} \\ &\quad - \frac{8}{30}g^{\mu_1\mu_2}g^{\mu_4\mu_5}\gamma^{\mu_3} + \frac{2}{30}g^{\mu_1\mu_4}g^{\mu_2\mu_5}\gamma^{\mu_3} - \frac{18}{30}g^{\mu_1\mu_5}g^{\mu_2\mu_4}\gamma^{\mu_3} \\ &\quad + \frac{17}{30}g^{\mu_1\mu_2}g^{\mu_3\mu_5}\gamma^{\mu_4} - \frac{18}{30}g^{\mu_1\mu_3}g^{\mu_2\mu_5}\gamma^{\mu_4} + \frac{17}{30}g^{\mu_1\mu_5}g^{\mu_2\mu_3}\gamma^{\mu_4} \\ &\quad - \frac{13}{30}g^{\mu_1\mu_2}g^{\mu_3\mu_4}\gamma^{\mu_5} + \frac{17}{30}g^{\mu_1\mu_3}g^{\mu_2\mu_4}\gamma^{\mu_5} - \frac{8}{30}g^{\mu_1\mu_4}g^{\mu_2\mu_3}\gamma^{\mu_5}\end{aligned}$$

$$\begin{aligned}\Gamma_{5'}^{\mu_1\mu_2\mu_3\mu_4\mu_5} &= +\frac{i}{3}g^{\mu_1\mu_2}\varepsilon^{\mu_3\mu_4\mu_5\beta}\gamma_\beta\gamma_5 - \frac{i}{2}g^{\mu_1\mu_3}\varepsilon^{\mu_2\mu_4\mu_5\beta}\gamma_\beta\gamma_5 + \frac{i}{6}g^{\mu_1\mu_4}\varepsilon^{\mu_2\mu_3\mu_5\beta}\gamma_\beta\gamma_5 \\ &\quad - \frac{i}{3}g^{\mu_1\mu_5}\varepsilon^{\mu_2\mu_3\mu_4\beta}\gamma_\beta\gamma_5 + \frac{i}{6}g^{\mu_2\mu_3}\varepsilon^{\mu_1\mu_4\mu_5\beta}\gamma_\beta\gamma_5 + \frac{i}{6}g^{\mu_2\mu_5}\varepsilon^{\mu_1\mu_3\mu_4\beta}\gamma_\beta\gamma_5 \\ &\quad + \frac{i}{6}g^{\mu_3\mu_4}\varepsilon^{\mu_1\mu_2\mu_5\beta}\gamma_\beta\gamma_5 - \frac{i}{2}g^{\mu_3\mu_5}\varepsilon^{\mu_1\mu_2\mu_4\beta}\gamma_\beta\gamma_5 + \frac{i}{3}g^{\mu_4\mu_5}\varepsilon^{\mu_1\mu_2\mu_3\beta}\gamma_\beta\gamma_5.\end{aligned}\quad (12)$$

We now introduce the conventional definitions for the gluon field-strength tensor $G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig_s[A_\mu, A_\nu]$ and its dual $\tilde{G}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}$. After some rearrangements based on the identities $[P_\mu, P_\nu] = ig_s G_{\mu\nu}$ and $\varepsilon_{\mu\nu\alpha\beta}P^\alpha P^\beta = ig_s \tilde{G}_{\mu\nu}$, we come to the important result:

$$\begin{aligned}
S_2 &= \frac{-ig_s}{2(4\pi)^2} M \left(N_\varepsilon - \ln \frac{M^2}{\mu^2} \right) G_{\mu\nu} \sigma^{\mu\nu} \\
S_3 &= \frac{-ig_s}{3(4\pi)^2} \left(N_\varepsilon - \ln \frac{M^2}{\mu^2} \right) D^\nu G_{\mu\nu} \gamma^\mu \\
S_4 &= \frac{-ig_s^2}{6(4\pi)^2} \frac{1}{M} \left(G_{\mu\nu} G^{\mu\nu} + \frac{3}{2} i G_{\mu\nu} \tilde{G}^{\mu\nu} \gamma_5 - 3i G_\alpha^\mu G^{\nu\alpha} \sigma_{\mu\nu} - \frac{1}{2g_s} D_\alpha D^\alpha G_{\mu\nu} \sigma^{\mu\nu} \right) \\
S_5 &= \frac{-ig_s^2}{8(4\pi)^2} \frac{1}{M^2} \left(\frac{6}{5} i [D^\alpha G_{\mu\alpha}, G^{\mu\nu}] \gamma_\nu + \frac{2}{15} i [G_{\mu\alpha}, D^\alpha G^{\mu\nu}] \gamma_\nu \right. \\
&\quad \left. + \frac{4}{3} \left\{ D^\alpha G_{\mu\alpha}, \tilde{G}^{\mu\nu} \right\} \gamma_\nu \gamma_5 + \frac{2}{3} \left\{ G_{\mu\alpha}, D^\alpha \tilde{G}^{\mu\nu} \right\} \gamma_\nu \gamma_5 - \frac{8}{15g_s} D^\alpha D_\alpha D^\nu G_{\mu\nu} \gamma^\mu \right) \quad (13)
\end{aligned}$$

with $iD_\mu F \equiv [P_\mu, F]$ for any field F transforming like the $SU(3)_C$ adjoint representation.

The dominant penguin-induced corrections to the initial four-quark operator (1) are now explicitly obtained:

$$\mathcal{L}_{eff}^{gw} = -ig_w \sum_{n=2}^5 \bar{q}_1 \Gamma^A S_n \Gamma^B q_2 + \mathcal{O}(1/M^3). \quad (14)$$

Remarkably, trace and axial anomalous operators are in principle already generated at the lowest possible order in the momentum expansion, as inferred from the colour-singlet part of the scalar and pseudoscalar gluon densities contained in S_4^i .

Let us now apply these results to the case of the standard-model short-distance weak operator evolution.

3 Anomalies from standard-model operator evolution

Tree-level integration of the W gauge boson in $V - A$ charged current interactions leads to effective operators of the type (1) with $\Gamma^{A(B)} = \gamma_\mu^{(\mu)} (1 - \gamma_5)$.

Let us first focus on the well-suited case of charm quark loop contributions to Kaon decays. The effective coupling associated with the tree-level operator defined at the M_W scale

$$Q_2^{(c)} = 4(\bar{d}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu s_L) \quad (15)$$

is given by

$$g_w^{SM} = -V_{cs} V_{cd}^* G_F / \sqrt{2}, \quad (16)$$

with the notation $q_L \equiv \frac{1}{2}(1 - \gamma_5)q$.

The chiral structure of this operator obviously prevents the even n contributions from appearing. The first non-vanishing terms are thus obtained for $n = 3$ and $n = 5$. They correspond to dimension-six and -eight operators, respectively:

$$\mathcal{L}_{eff}^{\Delta S=1} = \mathcal{L}_{6d} + \mathcal{L}_{8d} + \dots \quad (17)$$

Still, it is worthwhile to analyse them since, as we shall see, they can also carry some strong anomaly effects.

Dimension-six operators

The dominant operators in the m_c^{-1} expansion (17) have been computed a long time ago [13]. Yet, we find it useful to illustrate how the external field formalism works in a simple case. It also allows us to fix our conventions. The effect of incomplete GIM mechanism [14] above m_c is at the next-to-leading level [15], i.e. beyond the scope of this paper. So, the leading short-distance evolution of $Q_2^{(c)}$ from M_W down to m_c only involves the usual current-current operators [16], which disappear once the charm quark is integrated out. Inserting the explicit form of S_3 in eq.(14), we obtain then at the leading order in the strong coupling constant:

$$\mathcal{L}_{6d} = \frac{G_F}{\sqrt{2}} V_{cs} V_{cd}^* \frac{g_s}{6\pi^2} \ln \left(\frac{m_c^2}{\mu^2} \right) \bar{d}_L D^\nu G_{\mu\nu} \gamma^\mu s_L \quad (18)$$

with $\mu < m_c$. Recall that the standard computation of dimension-six gluonic penguin operators involves the use of the classical equations of motion:

$$D^\nu G_{\mu\nu} = g_s \sum_{q=u,d,s} \left(\bar{q} \gamma_\mu \frac{\lambda^a}{2} q \right) \frac{\lambda^a}{2}. \quad (19)$$

Applying these equations on Lagrangian (18) together with the identity $(\lambda^a)_{ij} (\lambda^a)_{kl} = 2(\delta_{il}\delta_{kj} - \delta_{ij}\delta_{kl}/3)$ and the relevant Fierz reorderings, we indeed recover the well-known current-current and density-density colour-singlet operators:

$$\mathcal{L}_{6d} = \frac{G_F}{\sqrt{2}} V_{cs} V_{cd}^* \frac{\alpha_s}{4\pi} \ln \left(\frac{m_c^2}{\mu^2} \right) \sum_{k=3}^6 c_k^{(6)} Q_k^{(6)} \quad (20)$$

with

$$\begin{aligned} Q_3^{(6)} &= 4(\bar{d}_L \gamma_\mu s_L)(\bar{q}_L \gamma^\mu q_L) & Q_5^{(6)} &= 4(\bar{d}_L \gamma_\mu s_L)(\bar{q}_R \gamma^\mu q_R) \\ Q_4^{(6)} &= 4(\bar{d}_L \gamma_\mu q_L)(\bar{q}_L \gamma^\mu s_L) & Q_6^{(6)} &= -8(\bar{d}_L q_R)(\bar{q}_R s_L) \end{aligned}$$

and

$$c_3^{(6)} = c_5^{(6)} = -\frac{1}{9}, \quad c_4^{(6)} = c_6^{(6)} = \frac{1}{3}.$$

In our conventions, $q_R \equiv \frac{1}{2}(1 + \gamma_5)q$ and the light flavours $q = u, d, s$ are summed over.

Dimension-eight operators

The dimension-eight penguin-like operators are most easily obtained from the contribution of S_5 to eq.(14), which gives:

$$\mathcal{L}_{8d} = -\frac{G_F}{\sqrt{2}} V_{cs} V_{cd}^* \frac{\alpha_s}{4\pi} \frac{1}{m_c^2} \sum_{k=1}^5 c_k^{(8)} Q_k^{(8)} \quad (21)$$

with

$$\begin{aligned}
Q_1^{(8)} &= i\bar{d}_L [D^\alpha G_{\mu\alpha}, G^{\mu\nu}] \gamma_\nu s_L \\
Q_2^{(8)} &= i\bar{d}_L [G_{\mu\alpha}, D^\alpha G^{\mu\nu}] \gamma_\nu s_L \\
Q_3^{(8)} &= \bar{d}_L \left\{ D^\alpha G_{\mu\alpha}, \tilde{G}^{\mu\nu} \right\} \gamma_\nu s_L \\
Q_4^{(8)} &= \bar{d}_L \left\{ G_{\mu\alpha}, D^\alpha \tilde{G}^{\mu\nu} \right\} \gamma_\nu s_L \\
Q_5^{(8)} &= \bar{d}_L D^\alpha D_\alpha D^\nu G_{\mu\nu} \gamma^\mu s_L
\end{aligned}$$

and

$$c_1^{(8)} = \frac{6}{5}, \quad c_2^{(8)} = \frac{2}{15}, \quad c_3^{(8)} = -\frac{4}{3}, \quad c_4^{(8)} = -\frac{2}{3}, \quad c_5^{(8)} = -\frac{8}{15}g_s^{-1}.$$

It may be remarked that this effective Lagrangian is invariant under the *CPS* symmetry, as it should [17].

Dimension-eight operators for Kaon decays have already been considered by the authors of ref.[9]. Yet, their early use of the equations of motion introduces an explicit dependence on the light quark masses $m_{d,s}$, which is artificial as the ‘plunged’ propagator is fundamentally a function, not a differential operator acting on external spinors. Besides, their ensuing truncated expansion in $m_{d,s}$ is not allowed.

Notice that the last operator in eq.(21) is still linear in the gluon field-strength tensor. As such, it contributes to the finite part of the standard $\bar{d}s g$ one-loop diagram, providing thus an interesting check of our calculation. Indeed, the gluon momentum expansion of the ‘vacuum polarization’ integral that naturally arises from the four-quark approximation

$$\int_0^1 dx x(1-x) \ln \left(\frac{m_c^2 - x(1-x)q^2}{\mu^2} \right) = \frac{1}{6} \left(\ln \frac{m_c^2}{\mu^2} - \frac{1}{5} \frac{q^2}{m_c^2} \right) + \mathcal{O}(q^4/m_c^4) \quad (22)$$

displays the correct relative weight between the operators of eq.(18) and eq.(21) with $k = 5$.

Let us now investigate the anomalous content of Lagrangian (21). Operators possibly involving $G_a^{\mu\nu} G_{\mu\nu}^a$ and $G_a^{\mu\nu} \tilde{G}_{\mu\nu}^a$ gluon densities are necessarily contained in the combinations

$$Q_1^{(8)} + Q_2^{(8)} = i\bar{d}_L D^\alpha [G_{\mu\alpha}, G^{\mu\nu}] \gamma_\nu s_L \quad (23)$$

and

$$Q_3^{(8)} + Q_4^{(8)} = \bar{d}_L D^\alpha \left\{ G_{\mu\alpha}, \tilde{G}^{\mu\nu} \right\} \gamma_\nu s_L, \quad (24)$$

respectively.

Remarkably, the gluonic part of the first operator is a pure $SU(3)_C$ octet, and consequently cannot carry any trace anomalous effect, in contradistinction to the results of ref.[9].

On the other hand, the second operator does contain a $SU(3)_C$ singlet part. Indeed, dropping a harmless total derivative, eq.(24) can be rewritten:

$$Q_3^{(8)} + Q_4^{(8)} = \left[-i\bar{d}_L \overleftarrow{P}^{\dagger\alpha} \gamma_\nu \left\{ \frac{\lambda^a}{2}, \frac{\lambda^b}{2} \right\} s_L + i\bar{d}_L \left\{ \frac{\lambda^a}{2}, \frac{\lambda^b}{2} \right\} \gamma_\nu P^\alpha s_L \right] G_{\mu\alpha}^a \tilde{G}^{\mu\nu,b} \quad (25)$$

with $\overleftarrow{P}_\alpha^\dagger = -i\overleftarrow{\partial}_\alpha + g_s A_\alpha$, the derivative acting on the left. Taking then the trace over colour and Lorentz indices, which amounts to the substitution

$$G_{\mu\alpha}^a \tilde{G}^{\mu\nu,b} \rightarrow \frac{\delta^{ab}}{8} \frac{\delta_\alpha^\nu}{4} G_{\rho\sigma}^c \tilde{G}_c^{\rho\sigma}, \quad (26)$$

and including the coefficients $c_3^{(8)}$ and $c_4^{(8)}$, we come to the following axial anomalous operator:

$$Q_{AA}^{(8)} = \frac{-i}{12} \left[\bar{d}_L P s_L - \bar{d}_L \overleftarrow{P}^\dagger s_L \right] G_{\mu\nu}^a \tilde{G}_a^{\mu\nu}. \quad (27)$$

The quark classical equations of motion

$$P s = m_s s, \quad \bar{d} \overleftarrow{P}^\dagger = m_d \bar{d} \quad (28)$$

may eventually be used to put this result in a more conventional form. Yet, the matrix element of $G_{\mu\nu}^a \tilde{G}_a^{\mu\nu}$ between the vacuum and a neutral pion state requires isospin violation in the factorization approximation. The charm-induced axial anomaly effect on hadronic K decays is thus negligible as far as the $\Delta I = 1/2$ amplitude is concerned.

Dimension-five and -seven operators

We have seen that the chiral structure of the tree-level operator (15) prevents any S_{2n} contribution from surviving at the effective Lagrangian level. However, it is well-known that dimension-five ‘chromo-magnetic’ penguin operators do arise in the Standard Model once we go beyond the four-quark approximation. A full short-distance calculation with massive W propagation gives indeed for both the charm and top contributions [18]:

$$\mathcal{L}_{5d} = \frac{G_F}{\sqrt{2}} V_{q's} V_{q'd}^* \frac{g_s}{32\pi^2} c_{q'}^{(5)} Q^{(5)} \quad (q' = c, t) \quad (29)$$

with

$$Q^{(5)} = m_s \bar{d}_L G_{\mu\nu} \sigma^{\mu\nu} s_R + m_d \bar{d}_R G_{\mu\nu} \sigma^{\mu\nu} s_L$$

and

$$c_{q'}^{(5)}(m_{q'} \ll M_W) = \mathcal{O}\left(\frac{m_{q'}^2}{M_W^2}\right), \quad c_{q'}^{(5)}(m_{q'} \gg M_W) = \mathcal{O}(1).$$

So, the longitudinal part of the W propagator induces a chiral structure compatible with the S_{2n} operators once the light quark equations of motion are used. However, the price to pay is a suppression factor of the order of $m_s m_c / M_W^2$ or m_s / m_t . This can be seen explicitly from eqs.(29) and (13) in the case of S_2 . Still, similar suppressions are expected to occur also for higher S_{2n} structures in hadronic K decays. Indeed, the helicity flip undergone by external spinors implies a linear dependence on the light quark masses, while the denominators can be inferred from the limit $M_W \rightarrow \infty$ in the charm case and from dimensional arguments in the top case.

Let us now turn to hadronic B decays. In exactly the same way, the virtual top quark contribution is suppressed by a factor of the order of m_b / m_t . On the other hand, the charm loop contribution induced by the tree-level operator $Q_2^{(c)} = 4(\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)$ is rather troublesome since the momenta associated with gluon fields can now be larger than

m_c . For illustration, the time-like squared momentum of the single gluon configuration generated by the ‘vacuum polarization’ integral (22) is likely in the range of $m_b^2/4 \lesssim q^2 \lesssim m_b^2/2$ for two-body exclusive B decays [19]. If such is the case, the expansion in inverse powers of the charm mass obviously breaks down, non-local operators survive and an absorptive component may even arise for $q^2 \geq 4m_c^2$. Consequently, we cannot exclude non-negligible anomalous effects associated with S_{2n+1} chiral structures in $B \rightarrow \eta' K$, unless the factorization approximation is called for help.

Factorized amplitudes

The factorized part of the $B \rightarrow \eta' K$ decay amplitude reads:

$$\left\langle \eta' K \left| Q_2^{(c)} \right| B \right\rangle_F = -\frac{1}{3} \left\langle \eta' \left| \bar{c} \gamma_\mu \gamma_5 c \right| 0 \right\rangle \left\langle K \left| \bar{s} \gamma^\mu b \right| B \right\rangle. \quad (30)$$

The fact that heavy quarks can only contribute to light pseudoscalar decay constants when propagating in a loop implies:

$$\left\langle \eta' \left| \bar{c} \gamma^\mu \gamma_5 c \right| 0 \right\rangle = \left\langle \eta' \left| -i \text{Tr} \left[(\not{P} - m_c)_{xx}^{-1} \gamma^\mu \gamma_5 \right] \right| 0 \right\rangle, \quad (31)$$

with the trace taken over both spinor and colour indices. Inserting the first few terms of the charm quark propagator expansion (13) in the above matrix element, we come to the conclusion that only part of S_5 survives to give:

$$\left\langle \eta' \left| \bar{c} \gamma^\mu \gamma_5 c \right| 0 \right\rangle = \left\langle \eta' \left| \frac{\alpha_s}{12\pi} \frac{1}{m_c^2} \left[2 (D^\alpha G_{\beta\alpha})^a \tilde{G}_a^{\beta\mu} + G_{\beta\alpha}^a (D^\alpha \tilde{G}^{\beta\mu})_a \right] \right| 0 \right\rangle + \mathcal{O}(1/m_c^4). \quad (32)$$

While this quantity remains to be estimated, it is already interesting to notice that the hadronization of the $\bar{c}c$ pair into the η' meson proceeds indeed through S_{2n+1} gluon structures, unlike what emerges from ref.[20] where the charm quark equation of motion is used from the start.

The axial anomaly part of eq.(32) is given by:

$$\left\langle \eta' \left| \bar{c} \gamma^\mu \gamma_5 c \right| 0 \right\rangle_{AA} = \frac{1}{32m_c^2} \left\langle \eta' \left| \frac{\alpha_s}{\pi} \partial^\mu \left(G_{\alpha\beta}^a \tilde{G}_a^{\alpha\beta} \right) \right| 0 \right\rangle \equiv -i f_{\eta'}^{(c)AA} p_{\eta'}^\mu \quad (33)$$

with

$$f_{\eta'}^{(c)AA} = \frac{-1}{32m_c^2} \left\langle \eta' \left| \frac{\alpha_s}{\pi} G_{\alpha\beta}^a \tilde{G}_a^{\alpha\beta} \right| 0 \right\rangle \simeq -2 \text{ MeV}. \quad (34)$$

The QCD sum rules calculation of ref.[21] has been used to estimate the last matrix element. Consequently, the charm quark induced axial anomaly can only account for a few per cent of the experimental $B \rightarrow \eta' K$ decay amplitude in the factorization approximation.

Note that charm quark loop effects in factorized $B \rightarrow f_0(\sigma) K$ decay amplitudes may be treated in a similar way. Yet these appear to be even more suppressed:

$$\left\langle f_0(\sigma) \left| \bar{c} \gamma^\mu c \right| 0 \right\rangle = \left\langle f_0(\sigma) \left| -i \text{Tr} \left[(\not{P} - m_c)_{xx}^{-1} \gamma^\mu \right] \right| 0 \right\rangle = \mathcal{O}(1/m_c^4). \quad (35)$$

4 Anomalies beyond standard-model operator evolution

The minimal non-standard operator leading to S_{2n} -related anomalous effects reads

$$Q = (\bar{d}_L c_R)(\bar{c}_L s_R) \quad (36)$$

in the case of hadronic K decays. This tree-level operator generically arises in multi-Higgs models [22]. In this framework, the following dimension-seven operators are directly obtained from S_4 :

$$\mathcal{L}_{7d} = -g_w \frac{\alpha_s}{24\pi} \frac{1}{m_c} \sum_{k=1}^4 c_k^{(7)} Q_k^{(7)} \quad (37)$$

with

$$\begin{aligned} Q_1^{(7)} &= \bar{d}_L G_{\mu\nu} G^{\mu\nu} s_R \\ Q_2^{(7)} &= i \bar{d}_L G_{\mu\nu} \tilde{G}^{\mu\nu} s_R \\ Q_3^{(7)} &= i \bar{d}_L G_\alpha^\mu G^{\nu\alpha} \sigma_{\mu\nu} s_R \\ Q_4^{(7)} &= \bar{d}_L D_\alpha D^\alpha G_{\mu\nu} \sigma^{\mu\nu} s_R \end{aligned}$$

and

$$c_1^{(7)} = 1, \quad c_2^{(7)} = \frac{3}{2}, \quad c_3^{(7)} = -3, \quad c_4^{(7)} = \frac{-1}{2} g_s^{-1}.$$

In particular, their anomalous content is given by:

$$Q_{AA+TA}^{(7)} = \frac{1}{6} \bar{d}_L s_R \left(G_{\mu\nu}^a G_a^{\mu\nu} + \frac{3}{2} i G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} \right) \quad (38)$$

if the coefficients $c_1^{(7)}$ and $c_2^{(7)}$ are included. However, the effective coupling constant g_w is in principle suppressed by the characteristic scale of the new physics involved.

In the factorization approximation, the trace and axial anomalous operators merged in eq.(38) are the only contributions of eq.(37) to hadronic K decay amplitudes. They can thus be directly inferred from the factorization of Q matrix elements. Indeed, the charm quark loop contribution in

$$\langle \pi^+ \pi^- | Q | \bar{K}^0 \rangle_F = \frac{1}{12} \langle \pi^+ \pi^- | \bar{c} c | 0 \rangle \langle 0 | \bar{d}_L s_R | \bar{K}^0 \rangle \quad (39)$$

$$\langle \pi^- \pi^0 | Q | K^- \rangle_F = \frac{1}{12} \langle \pi^0 | \bar{c} \gamma_5 c | 0 \rangle \langle \pi^- | \bar{d}_L s_R | K^- \rangle \quad (40)$$

is readily estimated using the ‘plunged’ propagator, provided we make the replacement $\gamma^\mu \rightarrow 1$ in eq.(35) and $\gamma^\mu \gamma_5 \rightarrow \gamma_5$ in eq.(31). Let us point out that this simply amounts to the well-known substitutions [23]:

$$m_c \bar{c} c \rightarrow \frac{-2}{3} \frac{\alpha_s}{8\pi} G_{\mu\nu}^a G_a^{\mu\nu} + \mathcal{O}(\mu^2/m_c^2) \quad (41)$$

$$m_c \bar{c} i \gamma_5 c \rightarrow \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} + \mathcal{O}(\mu^2/m_c^2) \quad (42)$$

for heavy quark contributions to light hadron matrix elements, providing thus another check of our central result (13).

5 Conclusion

We have proposed a simple way to systematically generate penguin-like operators from the inverse mass expansion of the heavy quark propagator plunged in QCD external fields. This expansion has been explicitly performed up to the fifth order. The result, expressed in a compact and comprehensible form, is also suitable for other applications like, as we have seen, the heavy quark contribution to light meson decay constants.

This formalism has allowed us to clarify the role of the strong axial and trace anomalous operators in hadronic weak decays involving c , b or t quark loops. Our results may be summarized as follows:

- The virtual c contribution to K decay amplitudes provides us with an ideal laboratory to study the trace anomaly effects since the scalar gluon density could softly convert into a pair of light pions. However, in the Standard Model, the short-distance rise of the dimension-seven trace and axial anomalous operators is necessarily $1/M_W^2$ -suppressed due to the chiral structure of the four-quark effective weak interactions, while it is shown that there is no trace anomaly effect from dimension-eight operators. Besides, from factorization arguments, we conjecture a small impact of c quark induced strong anomalies on hadronic B decays. Notice that a complementary conclusion has been reached recently by the authors of ref.[24] for the axial ‘anomaly tail’ due to highly virtual gluons, i.e. above the m_b scale.
- The formalism applies to b quark loop effects in D meson decays too. However, these suffer in any case from a multi-Cabibbo suppression.
- The t quark induced anomalous operators, on the other hand, require to go beyond the four-quark approximation. Yet, a simple dimensional argument leads us to conclude that the chiral suppression in $K(B)$ decays is now controlled by the $m_{s(b)}/m_t$ ratio.

Consequently, if the strong axial and trace anomalies have a role to play in hadronic weak decays, in particular in $B \rightarrow \eta' K_S$ and $K \rightarrow \pi\pi$, they should arise from either nonperturbative effects induced by the light s , d and u quarks or new physics beyond the Standard Model.

Our formalism also provides us with a systematic classification of charm-induced operators for K decays. In particular, a complete set of gauge-invariant dimension-eight operators is displayed. Their effect, of the order of m_K^2/m_c^2 , might compete with other non-leading short-distance QCD corrections [15]. As such, they obviously deserve further attention, independently of their anomalous content. Let us notice that distinct $1/\mu^2$ effects beyond dimensional regularization have been discussed in ref.[25].

Finally, our method is readily extended to account for c quark contributions to radiative K decays. Indeed, the inclusion of photons simply proceeds through the substitutions:

$$\begin{aligned} g_s G_{\mu\nu} &\rightarrow g_s G_{\mu\nu} + eq F_{\mu\nu} \\ g_s D_{\alpha_1} \dots D_{\alpha_n} G_{\mu\nu} &\rightarrow g_s D_{\alpha_1} \dots D_{\alpha_n} G_{\mu\nu} + eq \partial_{\alpha_1} \dots \partial_{\alpha_n} F_{\mu\nu} \end{aligned}$$

with $F^{\mu\nu}$, the photon field-strength tensor and q , the heavy quark charge in units of the electron charge e .

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